EXTRANEOUS SOLUTIONS Because the domain of a logarithmic function generally does not include all real numbers, be sure to check for extraneous solutions of logarithmic equations. You can do this algebraically or graphically.

EXAMPLE 6 Standardized Test Practice

ELIMINATE CHOICES
Instead of solving the equation in Example 6 directly, you can substitute each possible answer into the equation to see whether it is a solution.

What is (are) the solution(s) of $\log 2x + \log (x - 5) = 2$?

Ⓐ $-5, 10$ Ⓑ 5 Ⓒ 10 Ⓓ $5, 10$

Solution

$\log 2x + \log (x - 5) = 2$	Write original equation.
$\log [2x(x - 5)] = 2$	Product property of logarithms
$10^{\log [2x(x-5)]} = 10^2$	Exponentiate each side using base 10.
$2x(x - 5) = 100$	$b^{\log_b x} = x$
$2x^2 - 10x = 100$	Distributive property
$2x^2 - 10x - 100 = 0$	Write in standard form.
$x^2 - 5x - 50 = 0$	Divide each side by 2.
$(x - 10)(x + 5) = 0$	Factor.
$x = 10$ or $x = -5$	Zero product property

CHECK Check the apparent solutions 10 and -5 using algebra or a graph.

Algebra Substitute 10 and -5 for x in the original equation.

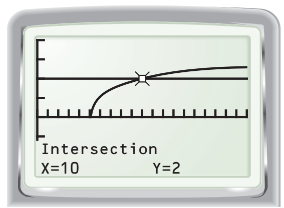
$$\log 2x + \log (x - 5) = 2$$
$$\log (2 \cdot 10) + \log (10 - 5) \stackrel{?}{=} 2$$
$$\log 20 + \log 5 \stackrel{?}{=} 2$$
$$\log 100 \stackrel{?}{=} 2$$
$$2 = 2 \checkmark$$

So, 10 is a solution.

$$\log 2x + \log (x - 5) = 2$$
$$\log [2(-5)] + \log (-5 - 5) \stackrel{?}{=} 2$$
$$\log (-10) + \log (-10) \stackrel{?}{=} 2$$

Because $\log (-10)$ is not defined, -5 is *not* a solution.

Graph Graph $y = \log 2x + \log (x - 5)$ and $y = 2$ in the same coordinate plane. The graphs intersect only once, when $x = 10$. So, 10 is the only solution.



▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 4, 5, and 6

Solve the equation. Check for extraneous solutions.

7. $\ln (7x - 4) = \ln (2x + 11)$ **3**

8. $\log_2 (x - 6) = 5$ **38**

9. $\log 5x + \log (x - 1) = 2$ **5**

10. $\log_4 (x + 12) + \log_4 x = 3$ **4**

Extra Example 6

What is (are) the solution(s) of $\log_6 3x + \log_6 (x - 4) = 2$? **D**

Ⓐ $-6, 2$
Ⓑ $-2, 6$
Ⓒ 2
Ⓓ 6

Key Question to Ask for Example 6

- Before starting, how can you tell that the given equation cannot have any solutions that are less than or equal to 5? **Because the domain of $y = \log_b x$ is $x > 0$, in this equation, you must have $2x > 0$ and $x - 5 > 0$, which means that $x > 5$.**

EXAMPLE 7 Use a logarithmic model

ASTRONOMY The *apparent magnitude* of a star is a measure of the brightness of the star as it appears to observers on Earth. The apparent magnitude M of the dimmest star that can be seen with a telescope is given by the function

$$M = 5 \log D + 2$$

where D is the diameter (in millimeters) of the telescope's objective lens. If a telescope can reveal stars with a magnitude of 12, what is the diameter of its objective lens?

ANOTHER WAY
For an alternative method for solving the problem in Example 7, turn to page 523 for the **Problem Solving Workshop**.

Solution

$M = 5 \log D + 2$	**Write original equation.**
$12 = 5 \log D + 2$	**Substitute 12 for *M*.**
$10 = 5 \log D$	**Subtract 2 from each side.**
$2 = \log D$	**Divide each side by 5.**
$10^2 = 10^{\log D}$	**Exponentiate each side using base 10.**
$100 = D$	**Simplify.**

▶ The diameter is 100 millimeters.

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Example 7

11. **WHAT IF?** Use the information from Example 7 to find the diameter of the objective lens of a telescope that can reveal stars with a magnitude of 7. **10 mm**

7.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 35, and 57
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 44, 47, 58, and 60
◆ = **MULTIPLE REPRESENTATIONS** Ex. 59

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: The equation $5^x = 8$ is an example of a(n) __?__ equation. **exponential**

2. ★ **WRITING** When do logarithmic equations have extraneous solutions? ***Sample answer:*** **When you have to factor to solve.**

EXAMPLE 1
on p. 515 for Exs. 3–11

SOLVING EXPONENTIAL EQUATIONS Solve the equation.

3. $5^{x-4} = 25^{x-6}$ **8**
4. $7^{3x+4} = 49^{2x+1}$ **2**
5. $8^{x-1} = 32^{3x-2}$ $\frac{7}{12}$
6. $27^{4x-1} = 9^{3x+8}$ $\frac{19}{6}$
7. $4^{2x-5} = 64^{3x}$ $-\frac{5}{7}$
8. $3^{3x-7} = 81^{12-3x}$ $\frac{11}{3}$
9. $36^{5x+2} = \left(\frac{1}{6}\right)^{11-x}$ $-\frac{5}{3}$
10. $10^{3x-10} = \left(\frac{1}{100}\right)^{6x-1}$ $\frac{4}{5}$
11. $25^{10x+8} = \left(\frac{1}{125}\right)^{4-2x}$ **−2**

Extra Example 7

The population of deer in a forest preserve can be modeled by the equation $P = 50 + 200 \ln (t + 1)$, where t is the time in years from the present. In how many years will be deer population reach 500? **about 8.5 yr**

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An **Animated Algebra** activity is available on-line for **Example 7**. This activity is also available on the **Power Presentations CD-ROM**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: Why do logarithmic equations sometimes have extraneous solutions?

- **Exponential equations can be solved either by equating exponents or by taking the logarithm of each side.**
- **Logarithmic equations can be solved either by applying the property of equality for logarithmic equations or by exponentiating each side.**

Logarithmic equations sometimes have extraneous solutions because the domain of every logarithmic function is $x > 0$, so any apparent solution that would lead to the logarithm of a nonpositive number must be rejected.